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An empirical formula for predicting the depth of hardness durign induction hardening

Parimal P. Shah
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AN EMPIRICAL FORMULA FOR PREDICTING THE DEPTH OF
HARDNESS DURING INDUCTION HARDENING

by

SHAH PARIMAL P.

A Thesis

Presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Master of Science

Lehigh University

1969

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

2/4/69
Date

George E. Lane
Professor in Charge

R. Gould
Chairman of the Department

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ABSTRACT

The prediction of the depth of hardness in terms of power density and scanning rate has been one of the central problems of interest in induction hardening of cylindrical parts like Axel Shafts, Piston Pins, Torsion Bars and King Pins.

In this thesis an empirical relationship is developed that can be used to predict the depth of hardness for AISI C 1045 normalized steel once the hardening conditions have been selected and without the necessity of conducting a trial hardening operation, thus saving electrical power, generator and scanning machine time.

The regression relationship developed in this thesis can be used, to predict the power density and hence the power by knowing the two variables namely depth of hardness and scanning rate on H. F. induction heating equipment. This type of information is extremely useful for the design of coils and the machines which use them.

In industrial practice of induction hardening the proper depth of hardness is obtained by a trial and error method.

The three main independent variables which affect the depth of hardness are (1) Frequency (2) Power Density and (3) Scanning Rate. With no exact prior information regarding

the variation in the depth of hardness due to the independent variables, six or eight pilot experiments are conducted.

These pilot experiments simulate the proper hardening conditions. After the experiment, the parts are checked for the depth of hardness. Out of these, one is chosen which closely represents the required depth of hardness, or near to the required depth of hardness.

This leads to the process standardization for a particular part. It was thought it might be possible to study the statistical relationship between the dependent and independent variables of this process. This study predicts the depth of hardness in terms of hardening conditions.

INTRODUCTION

This paper proposes to set forth some basic considerations and show by regression equation how controllable variables such as frequency, power density and scanning rate affect the depth of hardness of the heat treated part. To further simplify the subject matter it will be limited to the surface hardening of a cylindrical steel part, by scanning, primarily for producing a wear resistant surface but sometimes for other cogent reasons, e.g. an automotive axle shaft is a typical heat treated part. In an axle shaft it is necessary to surface harden the wear resistant surface for the bearing as well as surface harden the whole shaft for resisting bending and high torsional stresses.

PRINCIPLES OF INDUCTION HEATING

An induction heating circuit is fundamentally a transformer wherein the inductor carrying the alternating current is a primary and the substance to be heated is made the secondary by merely placing it within the confines of the loop formed by the inductor, there being no contact or connection between the two. The current flowing circumferentially through the inductor sets up magnetic lines of force in a circular pattern which thread through the

material being heated and induce a counter-circumferential flow of current therein.

Since the substance which carries the induced current is acting as a conductor, it also has an electrical resistance to this flow of energy. Thus we may compare induction heating to ordinary resistance heating and establish it likewise as that heat which is liberated as I^2R losses. That is to say, there is flow of current (I) and a resistance to this flow of current (R) which, combined, are responsible for the generation of heat.

The magnetic lines of force which induce the flow of energy are more concentrated at the midpoint of the width of the inductor and near its inside face. But the unusual characteristic of high frequency heating upon which all surface hardening applications depend is its tendency to concentrate on the surface of the conductor through which it flows.

This phenomenon, called "skin effect", is a function of frequency.

There are three independent variables which affect the depth of hardness. Let us consider them in detail.

(1) Frequency

We saw in principles of induction heating that the skin effect is a function of frequency. Other factors being equal, the higher the frequency, the shallower will be the depth of penetration of the heated zone. In our case the

depth of penetration (theoretical) can be varied by the frequency. The depth of penetration is related to the frequency in our experiment by the following formula⁽¹⁾

$$d = 3160 \sqrt{\frac{s}{\mu f}}$$

where

d = depth of penetration in inches

s = specific resistance

μ = relative magnetic permeability

f = frequency in cycles/sec.

When specific resistivity and relative magnetic permeability are substituted for steel, the formula reduces to

$$d = \frac{2.3}{\sqrt{f}}$$

The above formula is true for steel before it reaches the curie point.⁽²⁾ It can be seen that an appreciable change in high frequency (KC range) does not produce significant change in depth of penetration. The following example would make it more clear.

Frequency	Approximate Theoretical Depth of Penetration
220	0.0049
275	0.00439
280	0.00435
240	0.0047
300	0.0042

It should be noted that acceptable from a theoretical standpoint, the mathematical equations of frequency selection are misleading.

These equations which attempt to establish minimum frequencies for various work materials should be used with full realization of the bases of derivation and limitations.

As far as the experiment is concerned it can be said statistically that the effect of frequency on depth of penetration is insignificant compared to the effect of power density and scanning rate on the depth of hardness.

(2) Power Density

The expression of power density is defined as a measure of energy input to the inductor related to the surface area processed.

A long round can be surface hardened progressively, being heated as it goes through a relatively short induction coil and quenched as it emerges. Figure 5 shows such an arrangement.

If a limited amount of power is available, the long part may still be properly hardened to the required depth by introducing the power in a narrow coil.

In this instance, the depth of penetration is not a function of frequency but primarily of power density, and secondarily of the rate of travel through the inductors. (2)

Power density has a powerful effect on depth to which the part is heated.

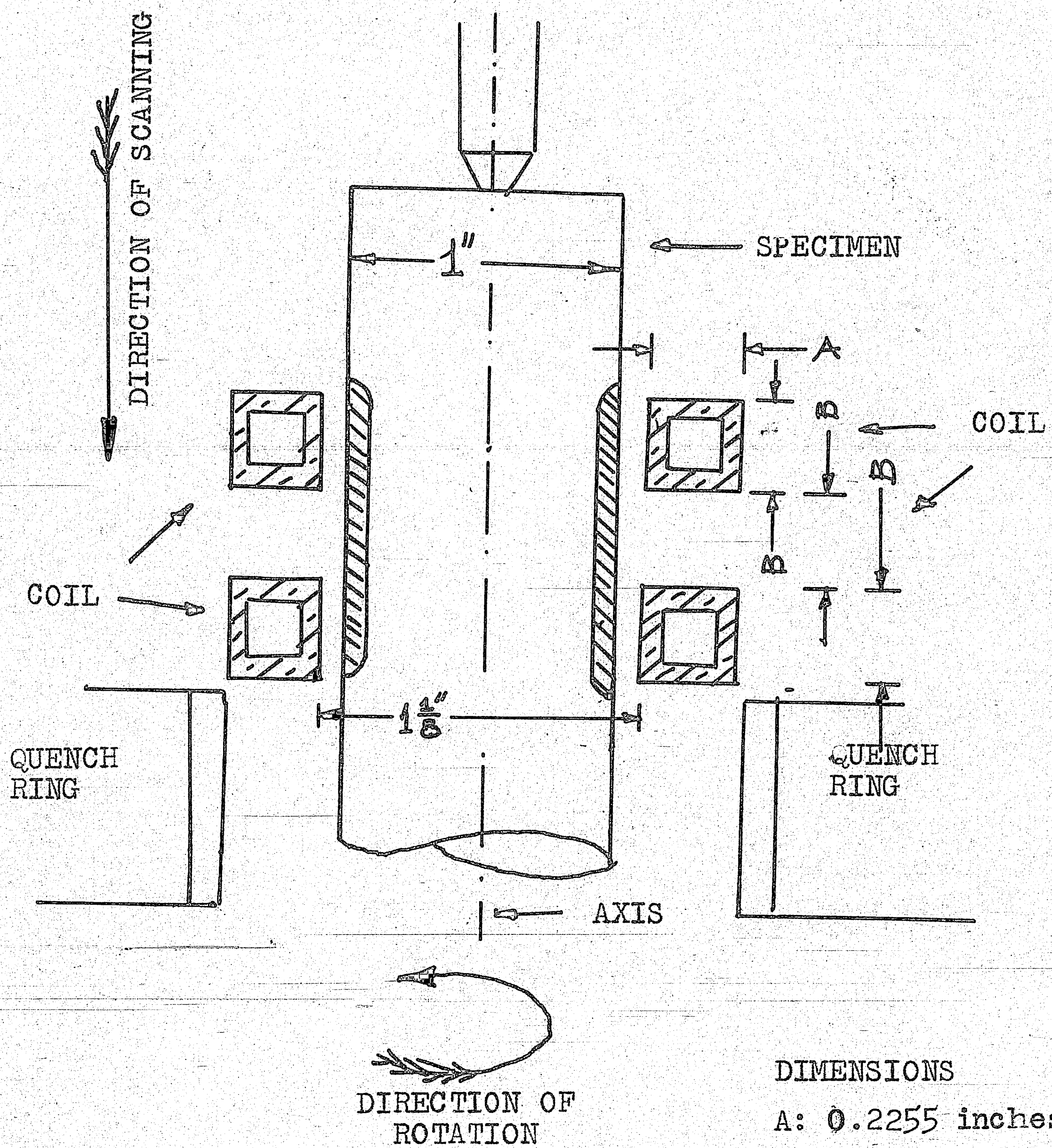


FIGURE 5

Graphs No. 1, 2, 3, 4 and 5 show the effect of varying power density on depth of hardness at a constant scanning rate.

(3) Scanning Rate

The last independent variable of the experiment is scanning rate.

The slower scanning rate gives a greater depth of hardness. By controlling the scanning rate we control the heat flow due to conduction. Slower scanning rate means a longer time interval between leaving the inductor and striking the quench and hence a greater heat flow inward by conduction. This results in appreciable change in the depth of hardness.

Graphs No. 6, 7, 8, 9 and 10 show such an effect at constant power density.

EXPERIMENTAL PROCEDURE

As previously stated, the purpose of this study is to determine an experimental method of calculating the depth of hardness. To obtain data for the study, the following hardening conditions were selected as being representative of those encountered in actual industrial practice.

A. Power Density - 8.12 KW/Sq. In., 14.86 KW/Sq. In.
22.3 KW/Sq. In., 27.5 KW/Sq. In.
and 32 KW/Sq. In.

B. Scanning Rate - 0.176"/sec., .224"/sec. 0.276"/sec.
0.299"/sec., 0.326"/sec.

Taking all possible combinations of the above conditions a total of 25 data points were obtained.

Data was collected using the Lepel P-42A scanning machine and Lepel H. F. generators. Longitudinal hardness was checked immediately after processing the sample to make sure the specimen was hardened according to its carbon content.

Metallographic tests were made to check the structure. The structure did not show decarburization. Figures No. 1, 2 and 3 show the metallographs of this experiment.

Hardness patterns across the diameter were measured by a Tukon hardness tester.

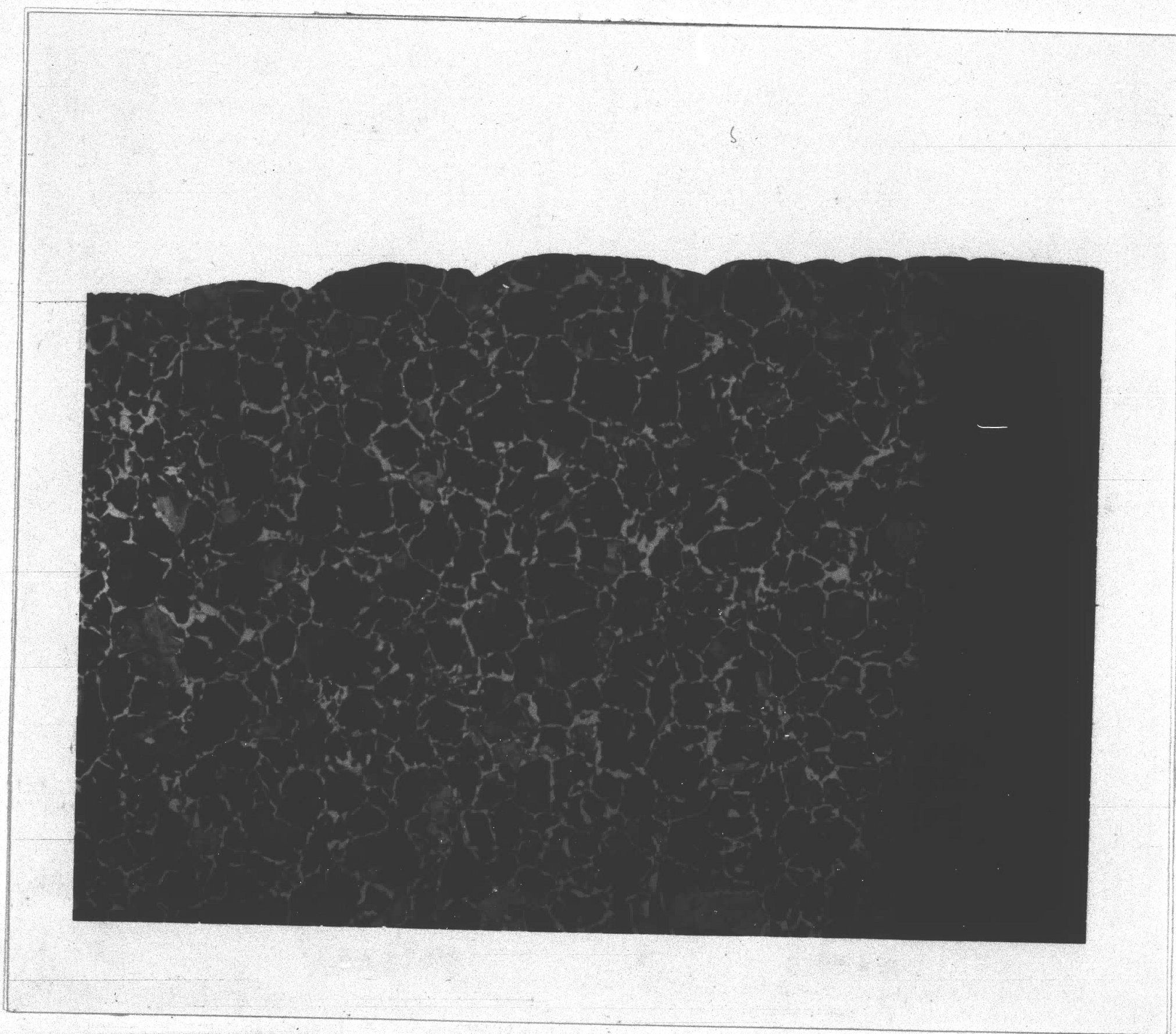
The sequence for taking the data was decided on as follows. Due to the difficulty involved in changing the

generator, all hardening conditions were completed for a given power density. The order of scanning conditions were selected with the help of the random number table.

Figure 1

METALLOGRAPH (75 X)

Metallograph showing no sign for decarburization.
This metallograph is compared with the metallograph
of core.



Relatively fine pearlitic areas (Dark)
Surrounded by ferrite at grain boundaries

Figure 2

Metallograph of core. This is compared with the
metallograph of edge. (See Figure 1)

CORE

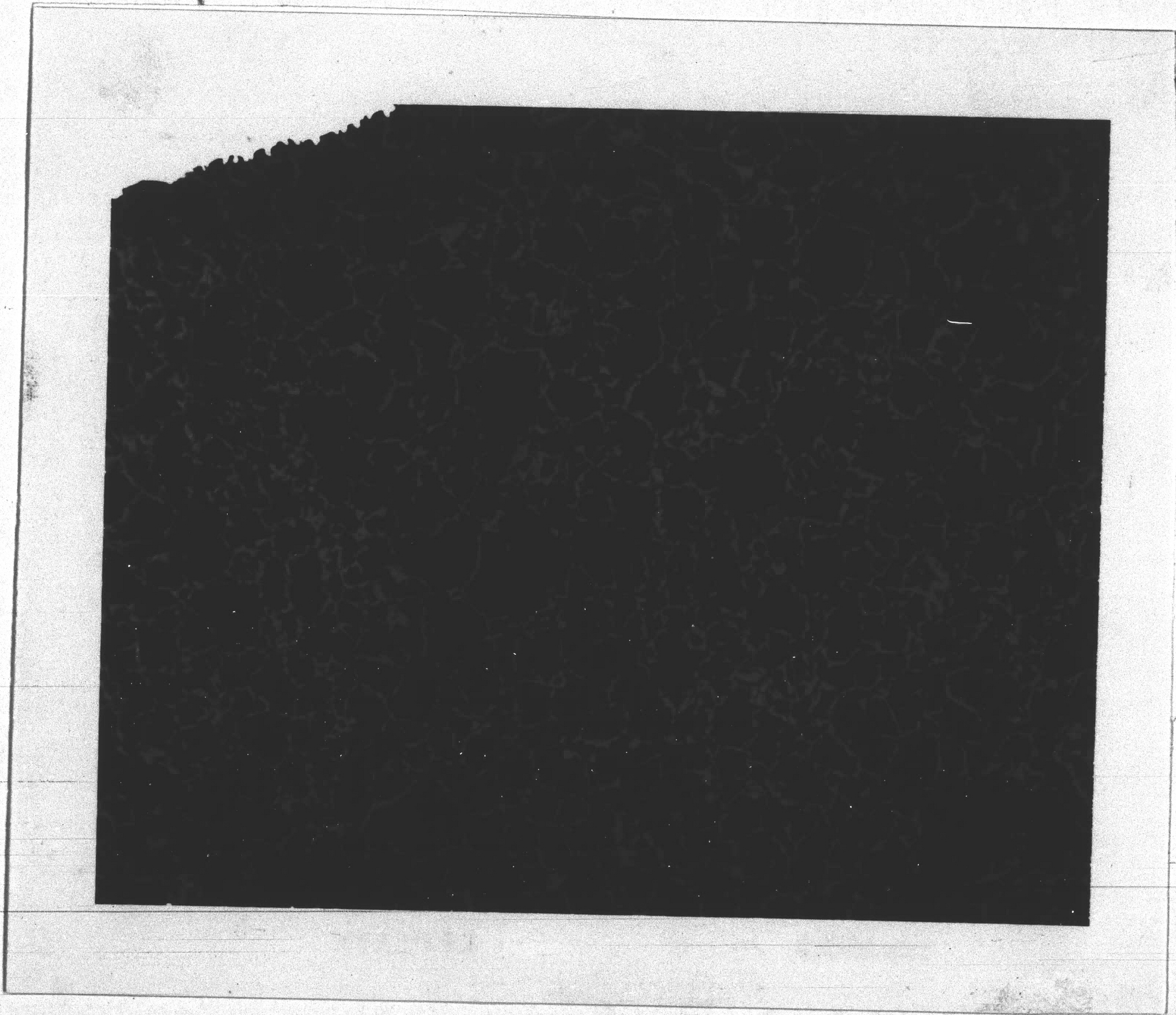


Figure 3

Metallograph at a higher magnification power showing relatively fine pearitic areas (Dark) surrounded by ferrite at grain boundries near the edge.

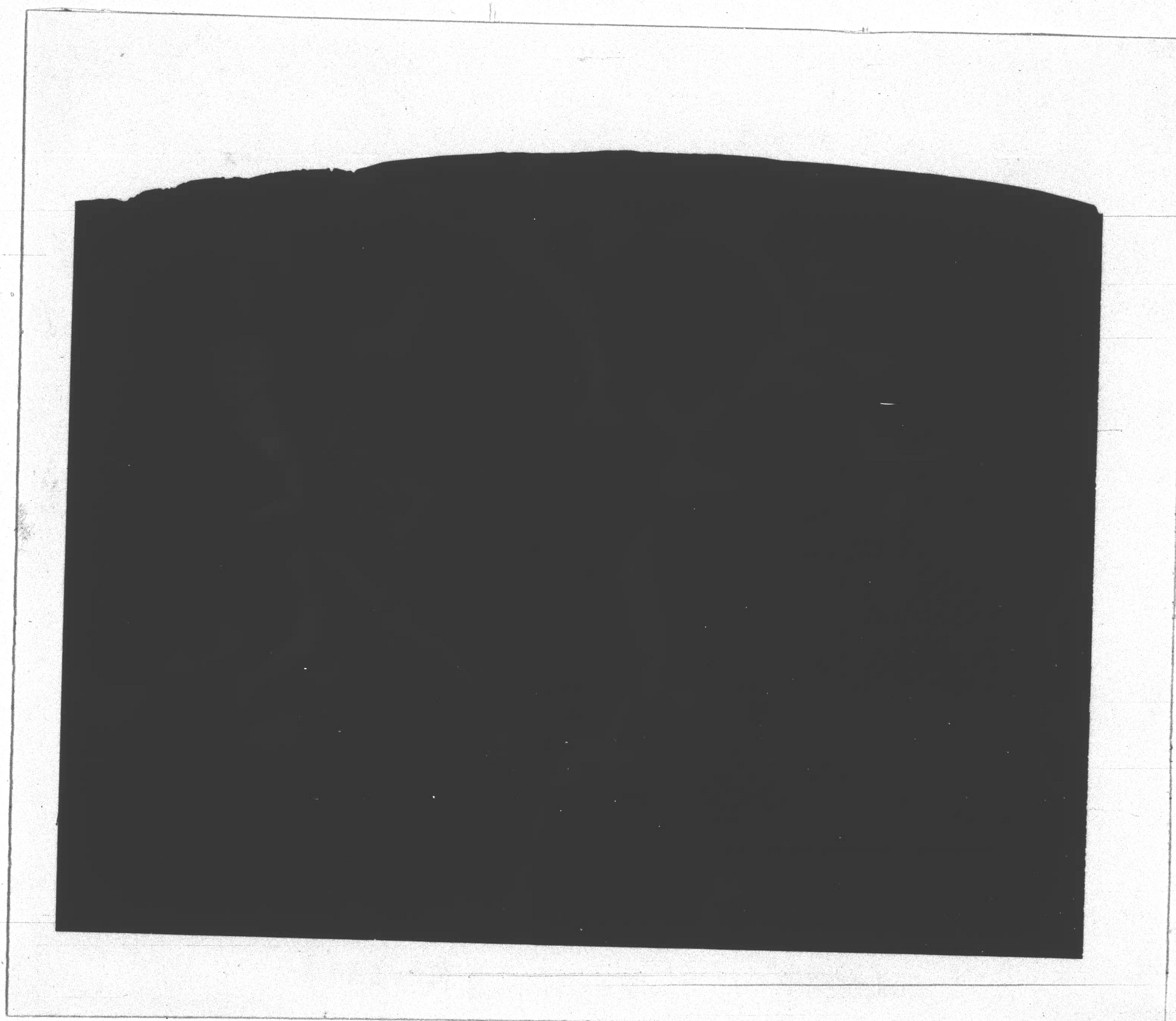


Figure 4

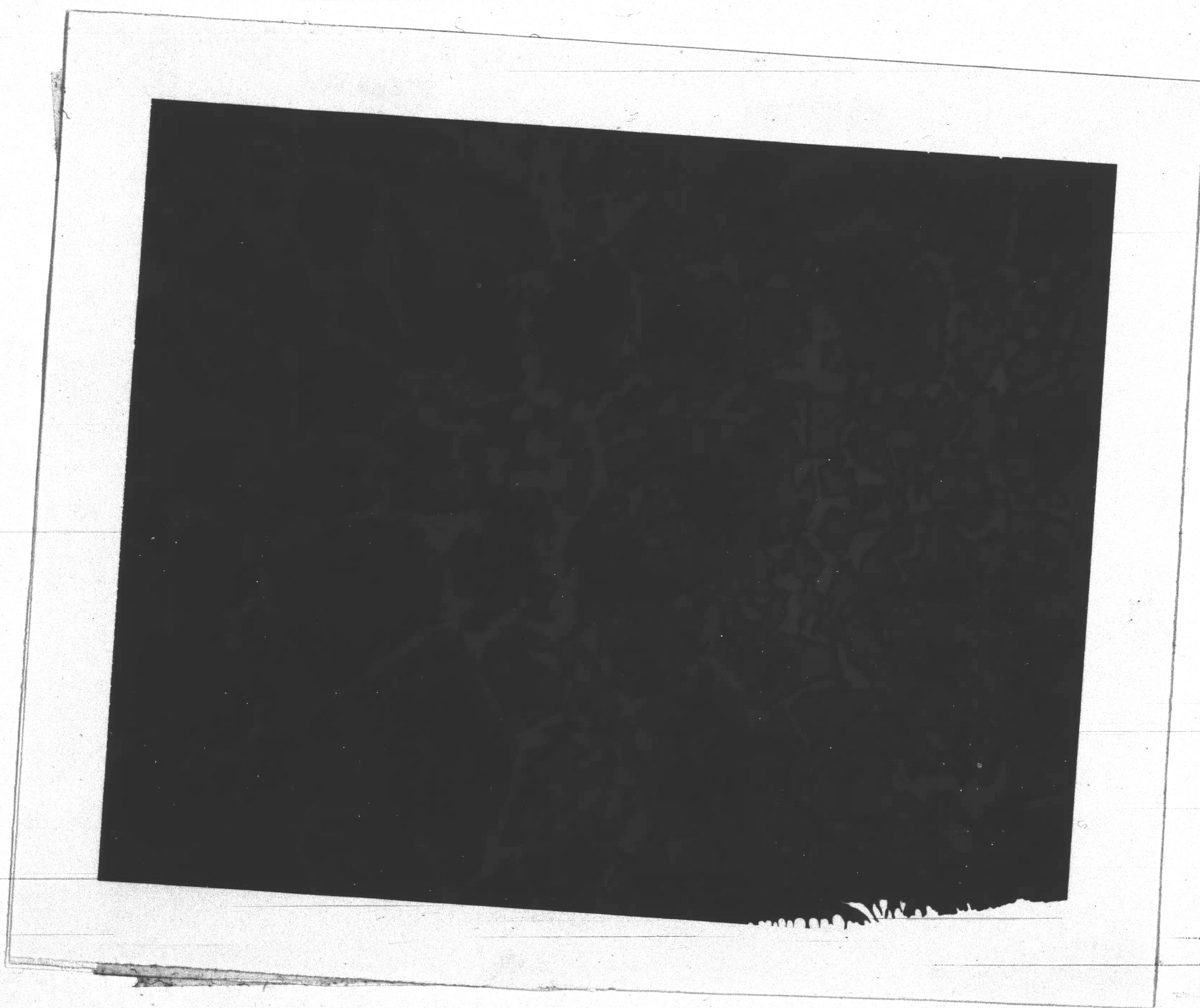
Metallograph (250 X)

Metallograph showing the core structure, Transition Zone, and hardened case.

FERRITE
&
PEARLITE
(CORE)

PEARLITE
&
MARTTENSITE
(TRANSITION ZONE)

MARTTENSITE
(CASE)



DISCUSSION AND RESULTS

The following relationship developed in this thesis can be used to determine the depth of hardness for a given set of hardening conditions if power density and scanning rate are known.

$$d_{1045} = 0.2755 - 0.10013 \log_{10}(P.D.) - 0.3775 \times (S.R.) \\ + 0.04889 \times (S.R.) \times \log_{10}(P.D.)$$

where

d = depth of hardness in inches

P.D. = Power density in Killowatt per square inch

S.R. = Scanning rate in inches per second.

To determine the effect of the independent variables on the depth of hardness, the results were plotted as a function of power density and scanning rate. These graphs are presented in appendix C. Linear regressions curves were fitted to each of the plots.

An examination of the graph revealed that the power density and scanning rate had an effect on the depth of hardness. This conclusion was later confirmed by analysis of variance.

The shape of the power density curve seemed to follow a log function. The graphs of power density were plotted on a semi-log paper. These points fell in a straight line.

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The shape of the power density curve seemed to follow a log function. The graphs of power density were plotted on a semi-log paper. These points fell in a straight line.

Statistically the power density graphs gave a good
fit.

ANALYSIS OF DATA

The results of the depth of hardness for the AISI C 1045 normalized steel were analysed using the statistical technique analysis of variance to test the significance of the independent variables. The results of the analysis are presented in appendix D.

The power density, scanning rate and the interaction terms were found to be significant at the 99% confidence level.

This confirmed statistically the conclusions drawn earlier from the examinations of the graphs of the depth of hardness plotted as a function of power density and scanning rate.

Having determined that the power density, scanning rate and interaction all have a significant effect on the depth of hardness, the next step was to develop an equation to predict the depth of hardness in terms of these independent variables.

The graphs (appendix C) showed that the effects of the independent variables on the depth of hardness were linear and the technique of multiple linear regression was used to develop the relationship.

The following equation for predicting the depth of hardness for AISI C 1045 normalized steel was developed:

$$d_{1045} = 0.2755 - 0.10013 \times \log_{10} (\text{P.D.}) - 0.3775 \times (\text{S.R.}) \\ + 0.04889 \times \text{S.R.} \times \log_{10} (\text{P.D.})$$

where

P.D. = Power density in Killowatt per square inch

S.R. = Scanning rate in inches per second

d = depth of hardness in inches.

The multiple linear regression analysis gave the following values:

- (1) The actual depth of hardness determined from the experiment
- (2) the value computed by regression equation, and
- (3) the difference between (1) and (2) which is known as residual.

Some of the values are given below and it can be seen that the difference is ± 0.004 . The F ratio in multiple linear regression also showed that the fit was significant.

Actual Value	Calculated by Regression	Difference (Residual)
0.125	0.12525	0.00025
0.112	0.11587	0.00387
0.082	0.0707	-0.00293
0.074	0.07138	-0.00262
0.085	0.08192	-0.00308
0.072	0.07425	0.00225
0.039	0.0478	0.00178

CONCLUSIONS

As stated previously the purpose of this study was to develop an empirical relationship for the determination of the depth of hardness. Based on the results of this study, the following conclusions were drawn:

1. The following relationship can be used to predict the depth of hardness for AISI C 1045 steel.

$$d_{1045} = 0.2755 - 0.10013 \times \log_{10} (\text{P.D.}) - 0.3775 \times (\text{S.R.}) \\ + 0.04889 \times \text{S.R.} \times \log_{10} (\text{P.D.})$$

This relationship can be used to predict the depth of hardness which will be encountered for a given set of hardening conditions without the necessity of first taking a trial hardening to determine any of the factors dependent on the results of the trial hardening.

2. The depth of hardness is dependent on the hardening conditions. The factors which have a significant effect on the depth of hardness are the power density and scanning rate. Interaction between power density and scanning rate contributes little, but it is of importance when the depth to be determined is precise. This is evidenced by the coefficient of the interaction term, it's maximum and minimum value.

The F ratio in the analysis of variance for power density and scanning rate are significantly great while the F-ratio for interaction barely makes 99% significant level.

3. This type of study should be used with proper understanding and may not be exactly true for other equipment.

4. As the depth of hardness increased, the transition zone became wider. When the depth of hardness was small there was a clear demarcation zone between Martensitic Structure (Case) and Ferritic and Pearlitic Structure (Core).

A typical demarcation zone is shown in Figure 4. This resulted in a different hardness pattern which can be seen on Graph No. 11.

AREAS FOR FUTURE STUDY

The techniques used, and the problems encountered in this study have brought to mind the following areas in which further study could be conducted. These areas include:

1. An expansion of studies of this type to include a greater variety of structures viz. Annealed, Quenched and Tempered.
2. A careful study and evaluation of regression equation for the different diameter for predicting the depth of hardness and correction factors, if any.
3. A detailed study of the nature of the hardness pattern under constant heating temperature as a function of diameter and material.

APPENDIX A

EQUIPMENT AND MATERIALS

List of Equipment

1. Model P-42A Lepel Scanning Machine
2. Lepel H. F. Generators
3. Rockwell and Tukon Hardness Tester
4. Polishing Machines
5. Frequency Meters.

Workpiece Material

1. C-1045 Normalized Steel - Bethlehem Steel Co.,
Heat No. 249-W-037

A. Diameter 1.00 inch

B. Hardness R_c 20

C. Length 7.0 inches

D. Chemical Composition

C	Mn	P	S	Si
0.47	0.92	0.02	0.048	0.15

APPENDIX B

DATA

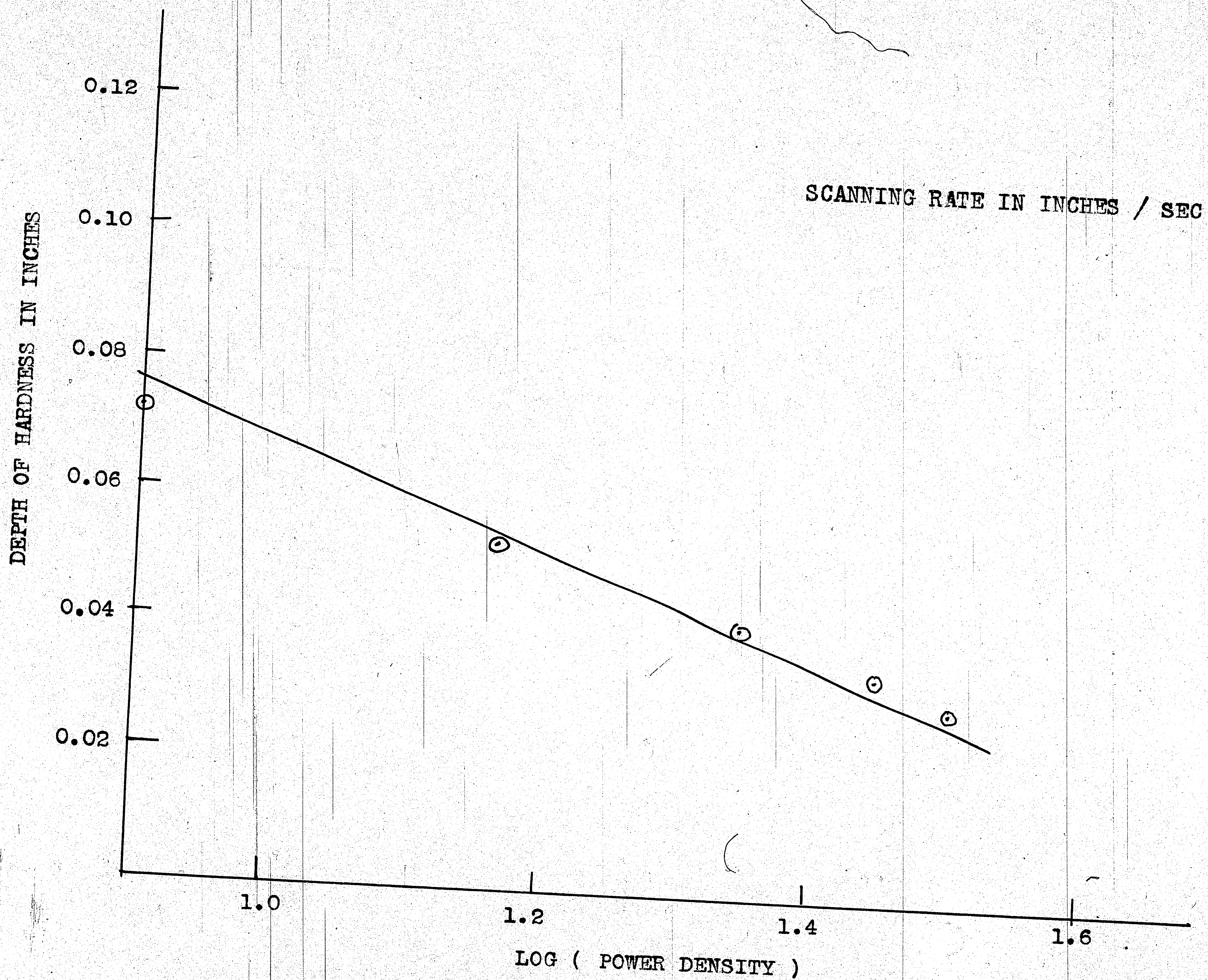
Frequency (KC)	Power Density (KW/sq.in.)	Scanning Rate (inches/sec.)	Depth of Hardness (inches)
220	8.12	0.176	0.127
			0.125
220	8.12	0.224	0.112
			0.110
220	8.12	0.276	0.094
			0.093
220	8.12	0.299	0.082
			0.085
220	8.12	0.326	0.074
			0.076
240	14.86	0.176	0.102
			0.104
240	14.86	0.224	0.088
			0.085
240	14.86	0.276	0.072
			0.070
240	14.86	0.299	0.062
			0.064
240	14.86	0.326	0.057
			0.055

Frequency (KC)	Power Density (KW/sq. in.)	Scanning Rate (inches/sec.)	Depth of Hardness (inches)
280	22.3	0.176	0.085
			0.086
280	22.3	0.224	0.072
			0.073
280	22.3	0.276	0.057
			0.055
280	22.3	0.299	0.047
			0.048
280	22.3	0.326	0.039
			0.041
275	27.5	0.176	0.074
			0.076
275	27.5	0.224	0.062
			0.064
275	27.5	0.276	0.047
			0.046
275	27.5	0.299	0.039
			0.041
275	27.5	0.326	0.030
			0.031
300	32	-	-

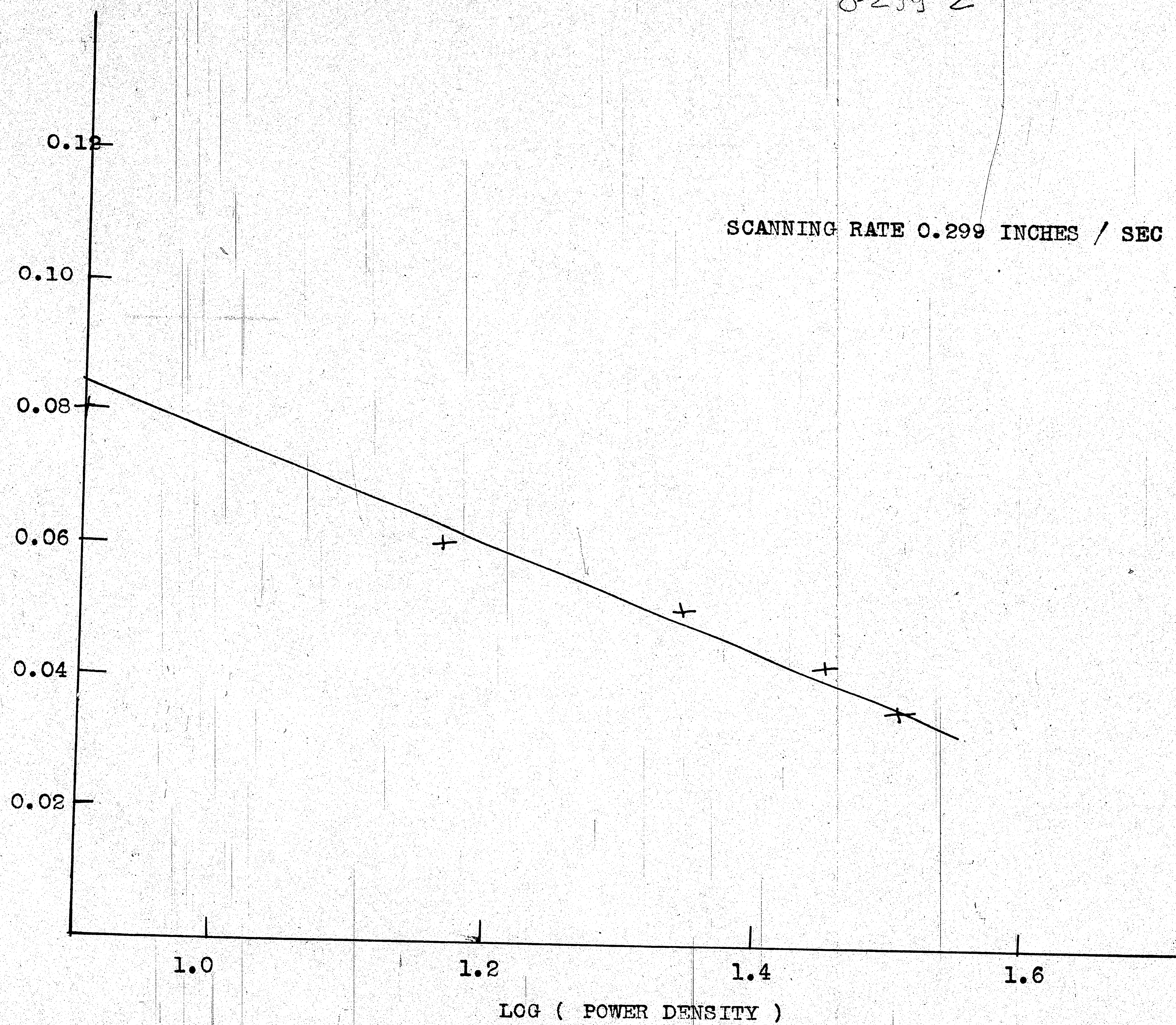
Frequency (KC)	Power Density (KW/sq.in.)	Scanning Rate (inches/sec.)	Depth of Hardness (inches)
300	32	0.224	0.057
			0.058
300	32	0.276	0.043
			0.042
300	32	0.299	0.035
			0.034
300	32	0.326	0.027
			0.029

The information contained in this report is based upon calculations or tests made by Lepel and is believed to be accurate, but no warranty or representation in reference thereto is intended or implied.

APPENDIX C



30
DEPTH OF HARDNESS IN INCHES



GRAPH 2

0.276 3

12
DEPTH OF HARDNESS IN INCHES

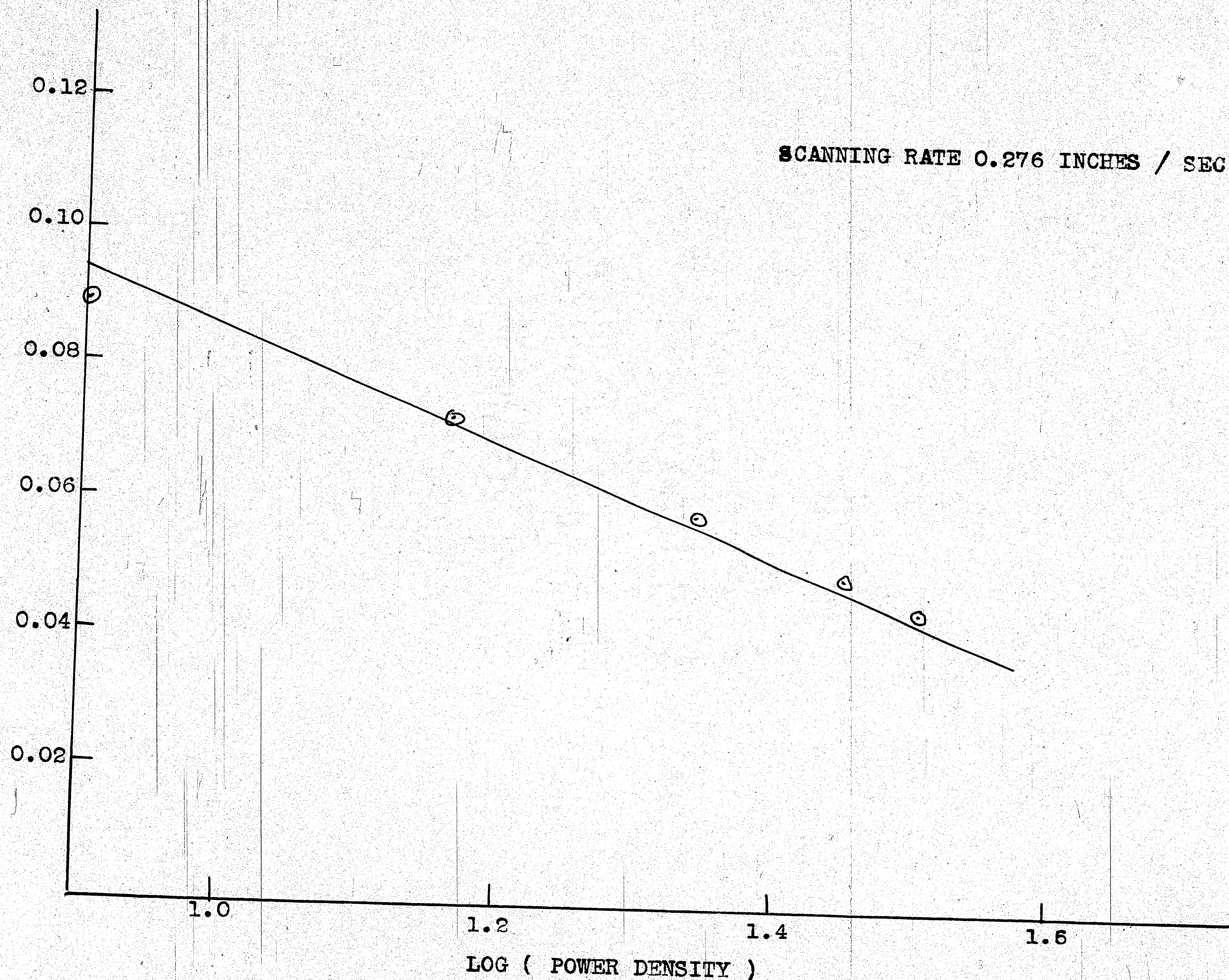


FIGURE 3

DEPTH OF HARDNESS IN INCHES

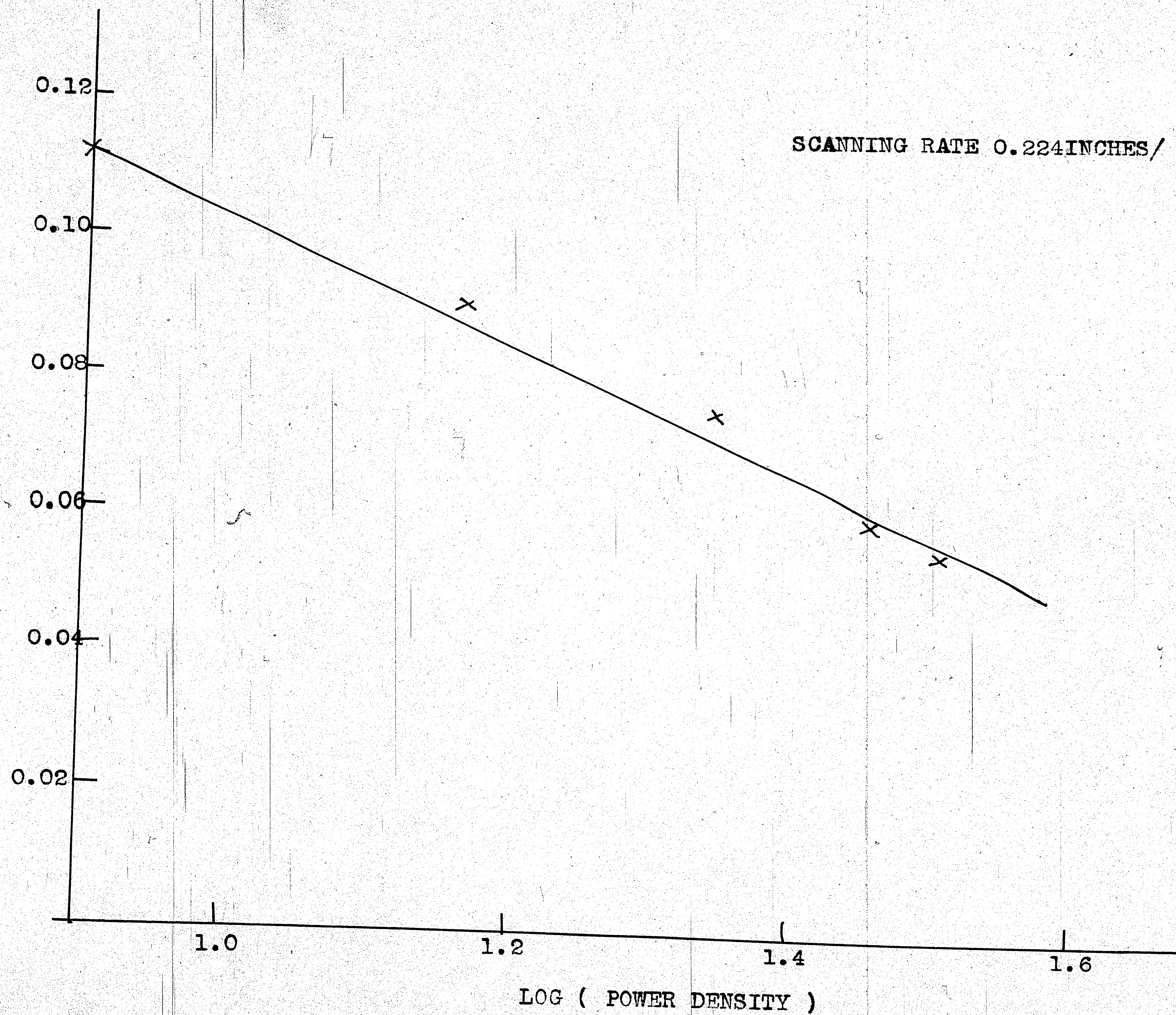


FIGURE 4

DEPTH OF HARDNESS IN INCHES

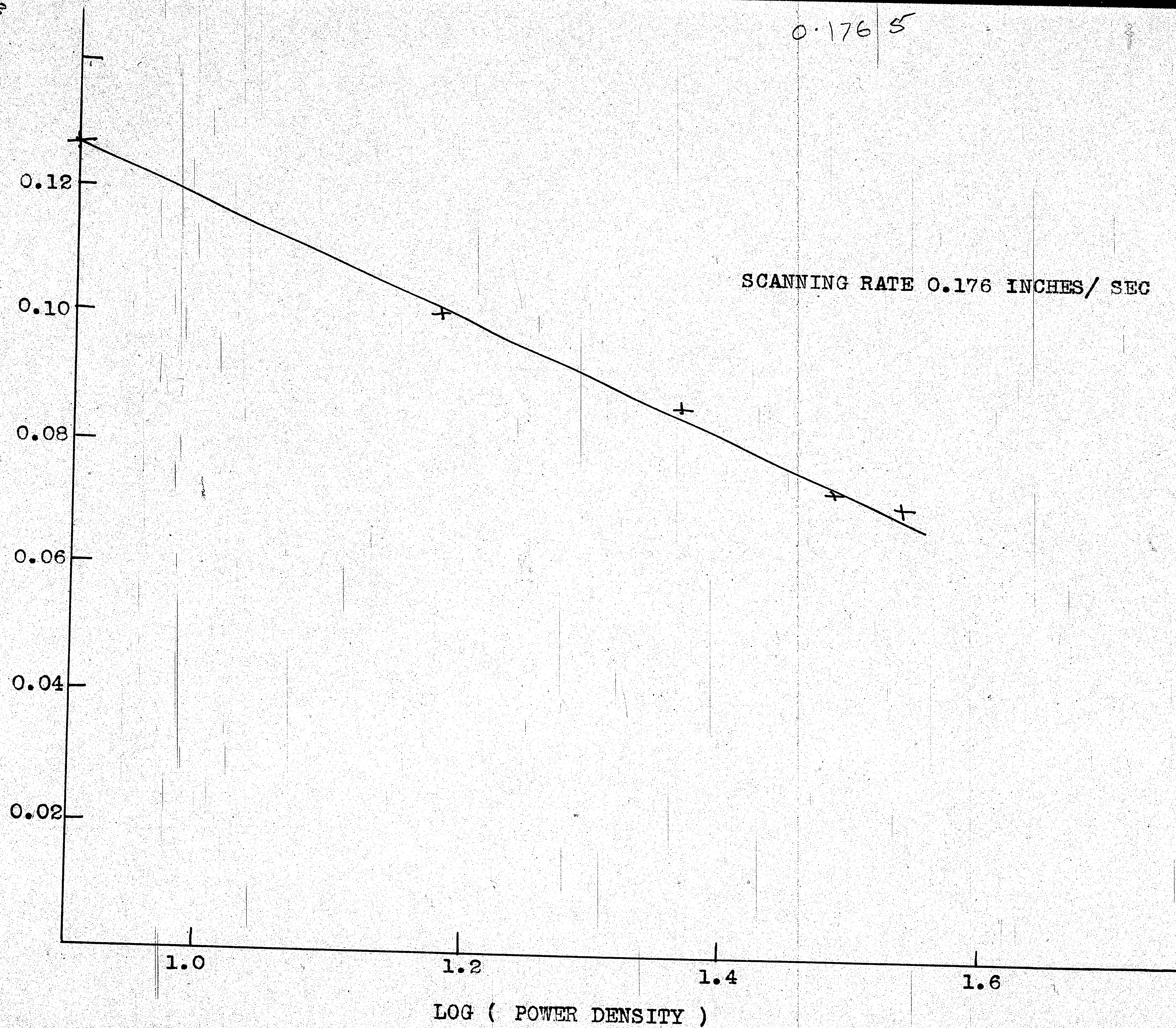


FIGURE 5

DEPTH OF HARDNESS IN INCHES

0.12
0.10
0.08
0.06
0.04
0.02

10

20

30

POWER DENSITY IN KW/ SQUARE INCH

SCANNING RATE
IN
INCHES/ SEC

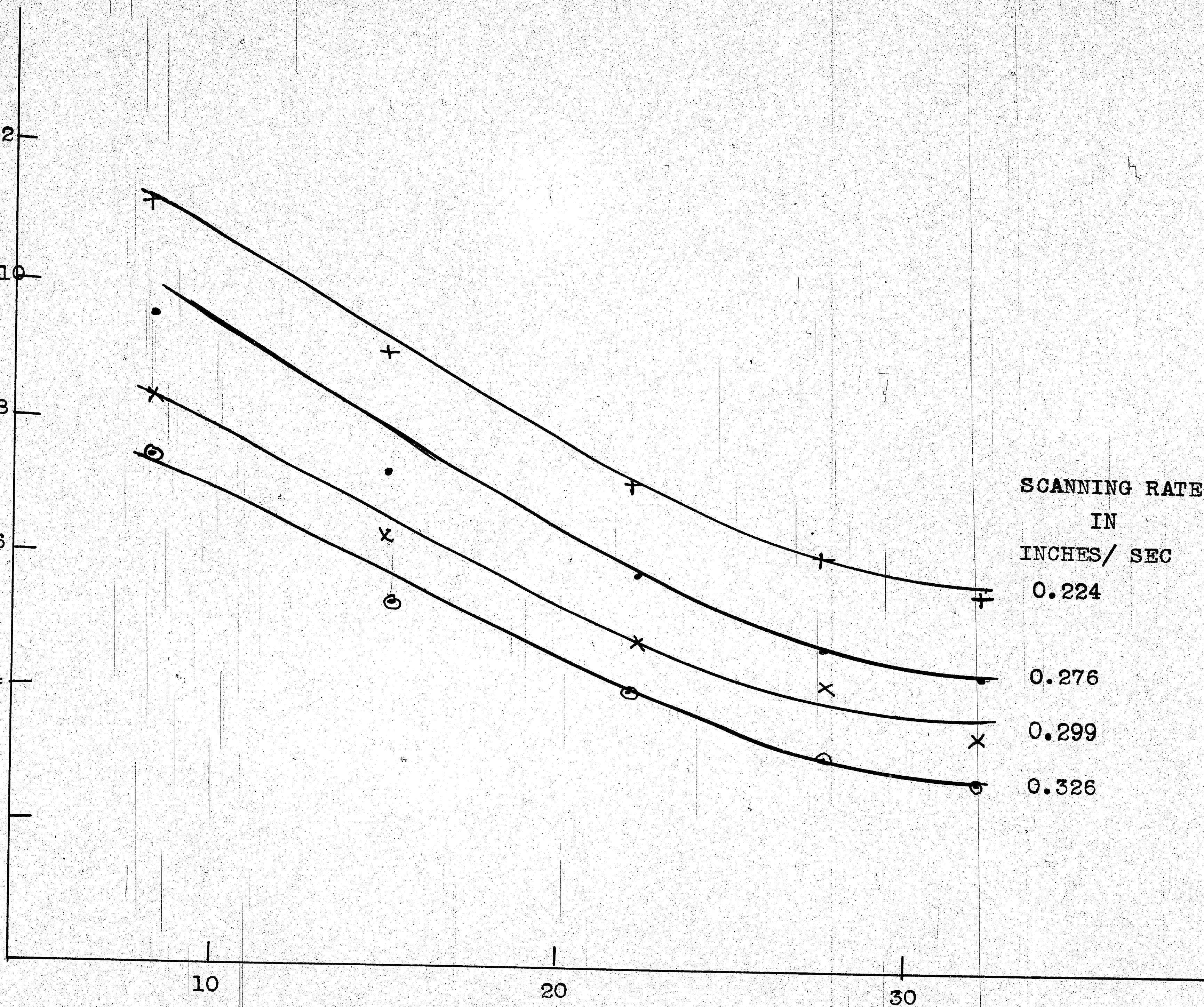
0.224

0.276

0.299

0.326

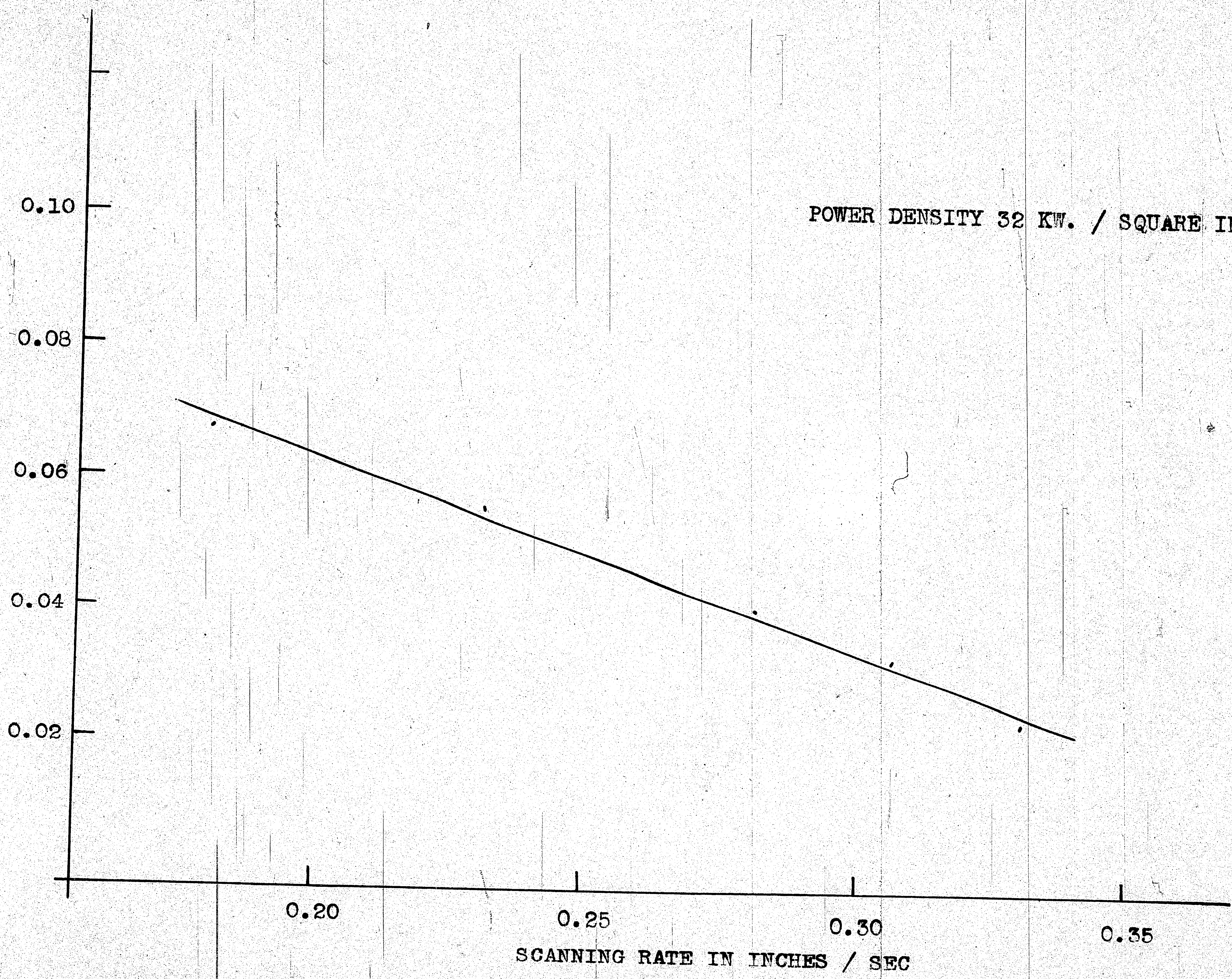
FIGURE 6



55

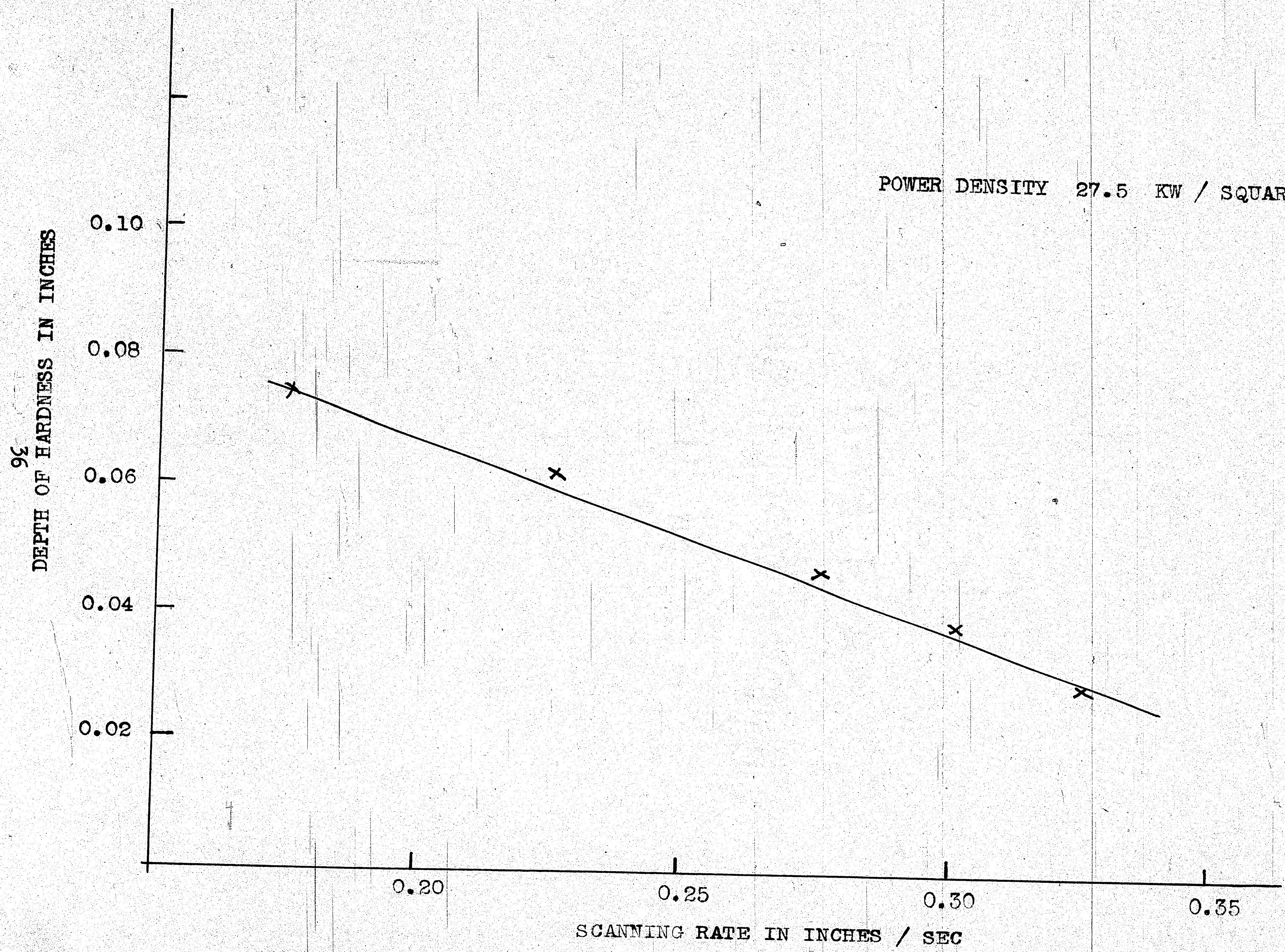
32 KW / \square " 7

55
DEPTH OF HARDNESS IN INCHES



GRAPH 7

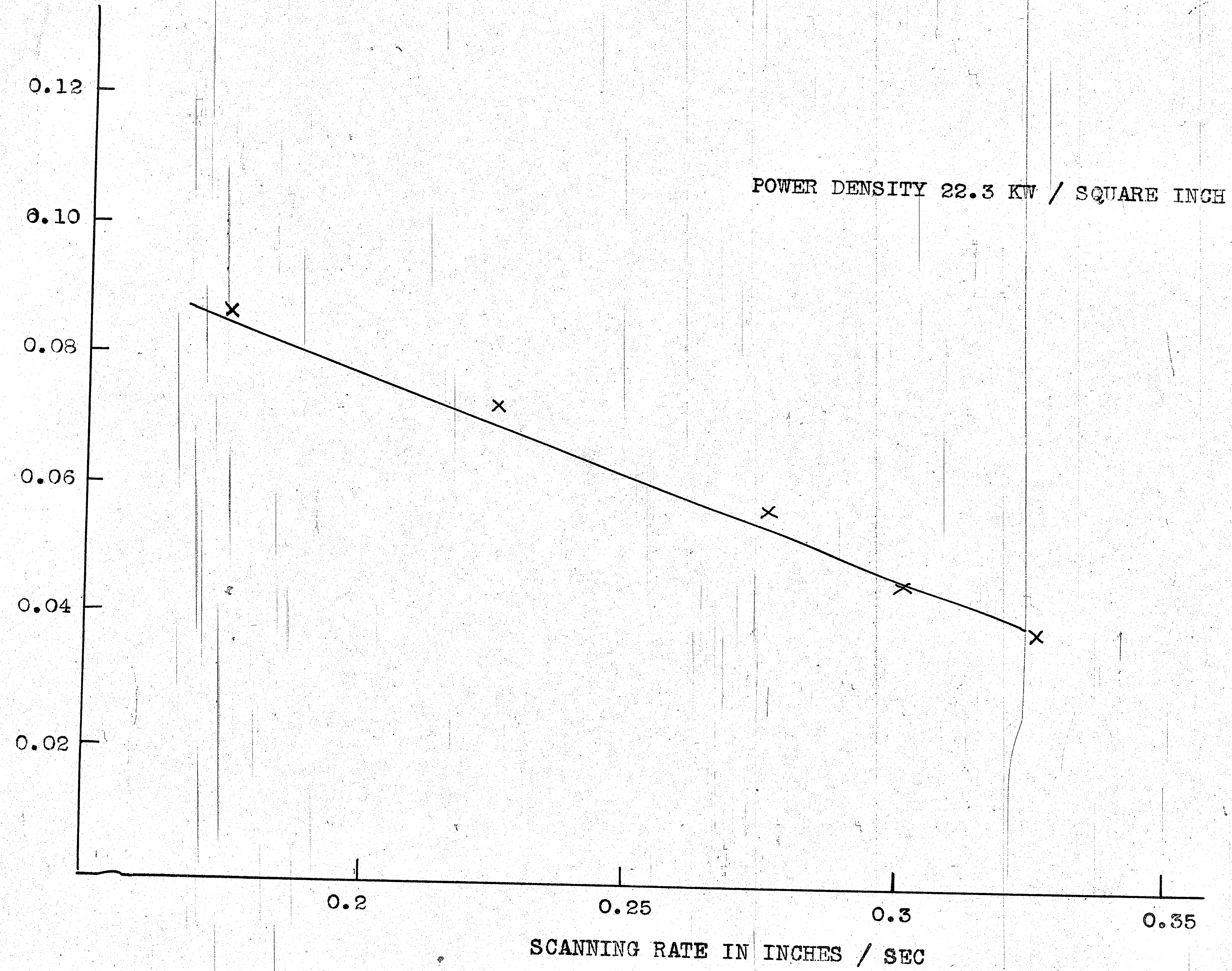
27.5 KW/IN² 8



GRAPH 8

22.3 kW/sq. In. 9

37
DEPTH OF HARDNESS IN INCHES

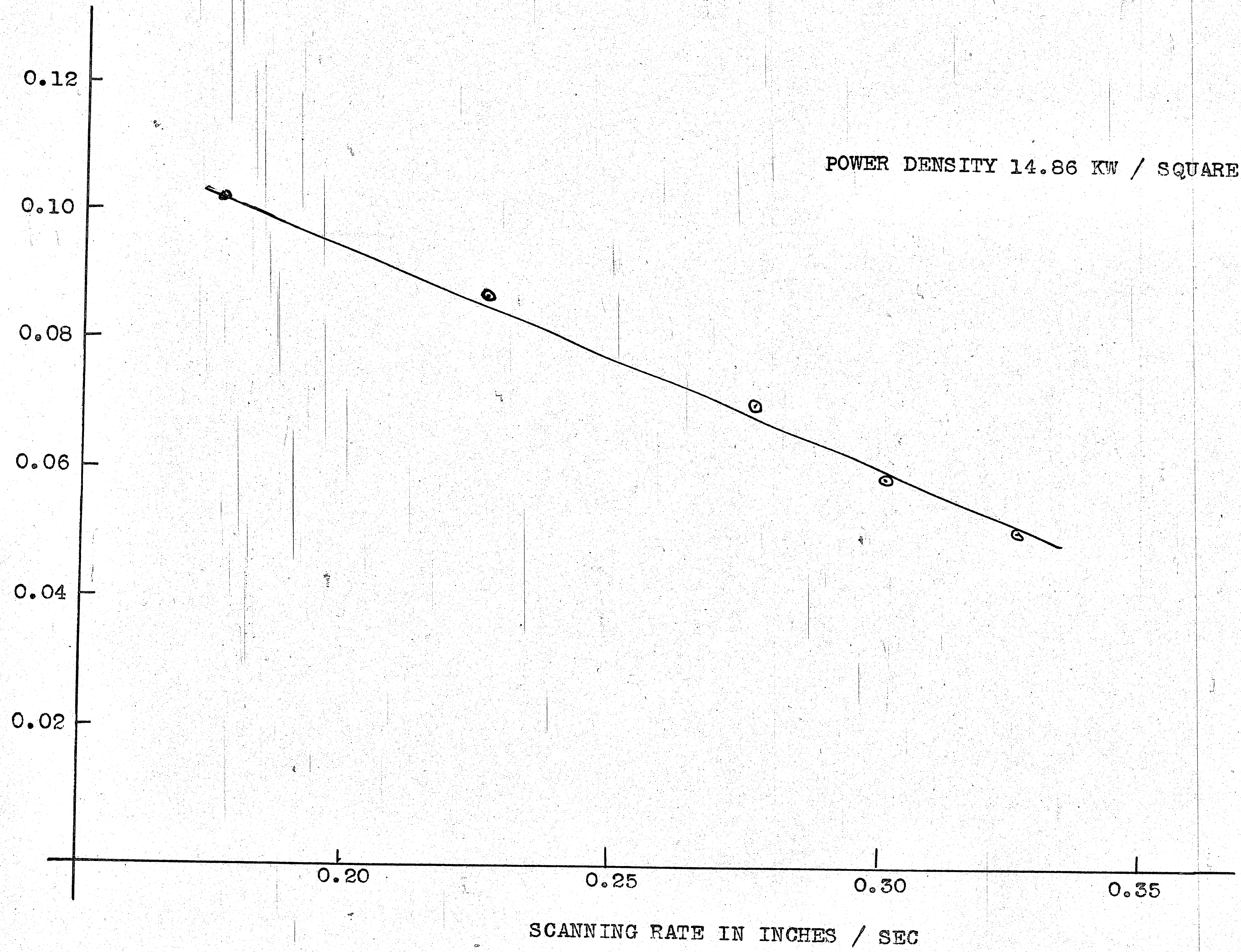


GRAPH 9

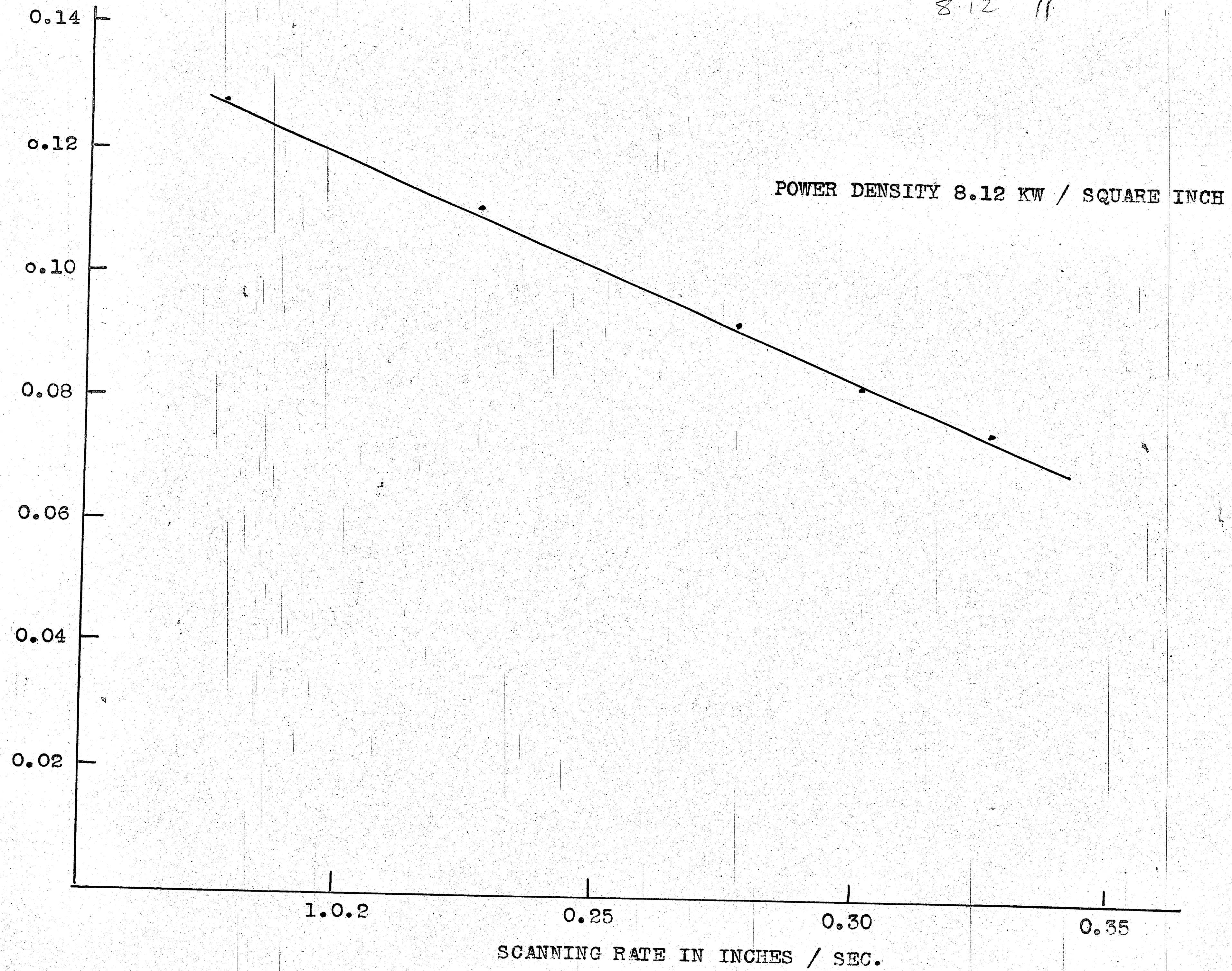
14.86 10

85
DEPTH OF HARDNESS IN INCHES

POWER DENSITY 14.86 KW / SQUARE INCH



GRAPH 10



VICKERS HARDNESS NUMBERS

LOAD 10 Kg.

200
300
400
500
600
700

.02

.03

.06

.08

.10

.12

.14

DISTANCE FROM SURFACE IN INCHES

①

②

③

④

⑤

①

②

③

④

⑤

ROCKWELL C HARDNESS EQUIVALENT

FIGURE 12

APPENDIX D

RESULTS OF ANALYSIS OF VARIANCE

DEPTH OF HARDNESS

MATERIAL AISI C 1045 Normalized Steel

Number of Factors	3
Number of Replicates	2
Levels of A (Power Density)	5
Levels of B (Scanning Rate)	5
Total No. of Observations	50

Effect	S.O.S.	D.O.F.	M.S.
** A	0.0132	4	0.0033
** B	0.0153	4	0.0038
* AB	0.0008	16	0.00005
RESIDUAL	0.0003	25	0.0000012

** Highly Significant

* Significant

F Ratios:	99%Level	95% Level	-
4/25	4.18	2.76	
16/25	2.85	2.09	

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- (3) Osborn, H. B., Jr., "High Power Radio Frequency Heating". Personal Communication January 16, 1968.

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Period

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